SEMINARY 8 (Solved Problems)

Wave superposition & standing waves

① Phasors

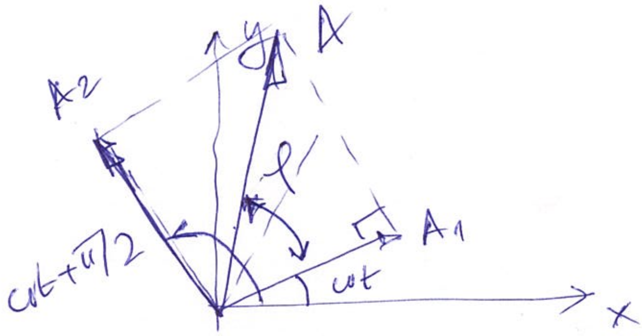
$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\varphi$$

$$\varphi_1 = 0 \quad \varphi_2 = \pi/2$$

$$\Rightarrow A^2 = A_1^2 + A_2^2 + \underbrace{2A_1A_2\cos\pi/2}_{0} = A_1^2 + A_2^2$$

$$A = \sqrt{9+16} = 5\,cm$$

$$y(t) = A\cos(\omega t + \varphi)$$

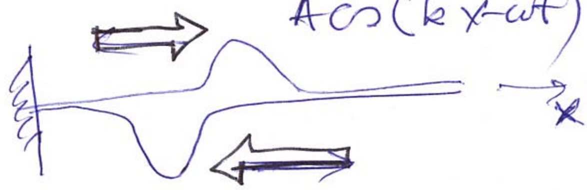


From the triangle $\Rightarrow$

$$tg\,\varphi = \frac{A_2}{A_1} = \frac{4}{3} \quad \Rightarrow \varphi = arc\,tg\,\frac{A_2}{A_1}$$

$$\Rightarrow \boxed{y(t) = 5\,cm\cos\left(\omega t + arctg\,\frac{4}{3}\right)}$$

② Standing wave obtained by superposition



of propagative:

incident: $y_1 = -A\cos(kx+\omega t)$

and reflected: $y_2 = A\cos(kx-\omega t)$ waves

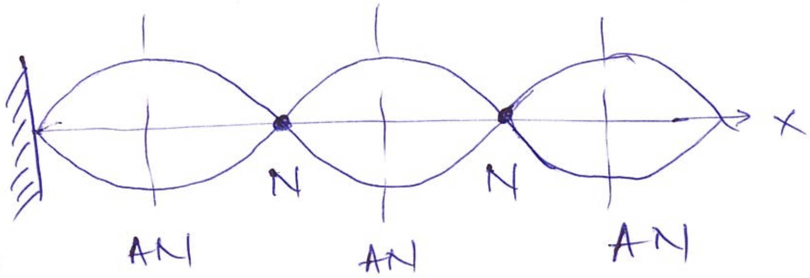
<u>Obs</u>: At reflection the phase changes by π

$$\Rightarrow A \rightarrow -A$$

$$y(x,t) = A\cos(kx-\omega t) - A\cos(kx+\omega t)$$
$$= A\left[\cos(kx-\omega t) - \cos(kx+\omega t)\right]$$
$$= -2A \sin\frac{kx-\cancel{\omega t}+kx+\cancel{\omega t}}{2} \sin\frac{\cancel{kx}-\omega t-\cancel{kx}-\omega t}{2}$$
$$= -2A \sin kx \sin(-2\omega t) = \qquad ! \ \boxed{\sin(-x) = -\sin x}$$

$$\boxed{y(x,t) = 2A \sin kx \sin(\omega t)}$$

Graphically:



- Nodes (N) $\quad \sin kx = 0 \Rightarrow kx = n\pi$
  $n = 0, 1, 2 \dots$

string not moving $\Rightarrow \boxed{x_n = \frac{n\pi}{k} = \frac{n\lambda}{2}}$ $\quad n = 0, 1, \dots$

$k = \frac{2\pi}{\lambda}$

The distance between two nodes:

$n=0 \quad x_0 = 0$
$n=1 \quad x_1 = \frac{\lambda}{2}$
$\Rightarrow \boxed{x_1 - x_0 = \frac{\lambda}{2}}$

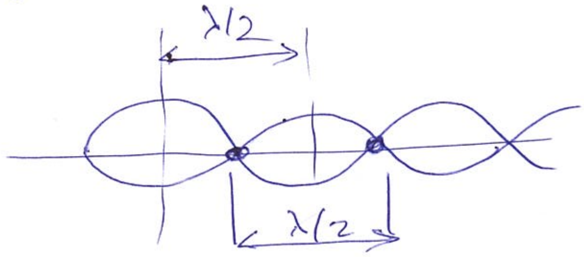
$x_{n+1} - x_n = \frac{\lambda}{2}$

- Antinodes: (AN) $\quad \sin Kx = \pm 1 \quad Kx = (n+1)\pi/2$
  (maxima) $\quad n = 0, 1 \dots$

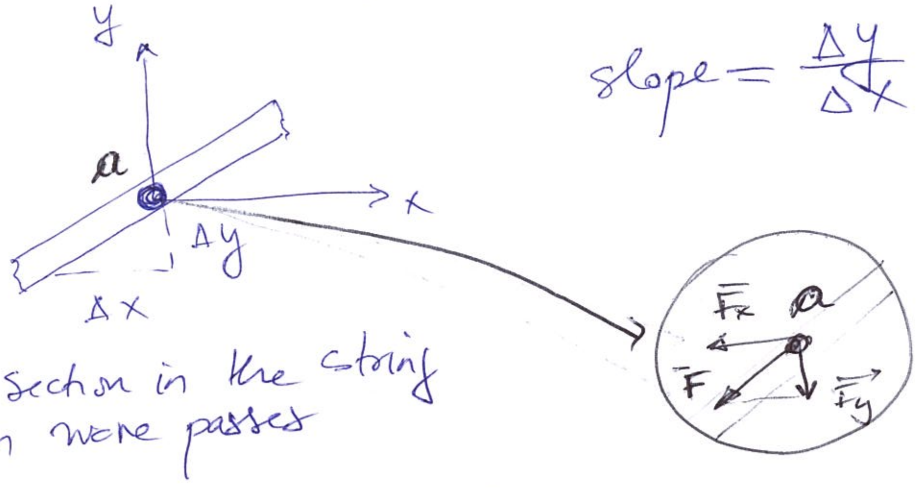
$\Rightarrow x_n = (n+1)\lambda/4$

$n=0 \quad x_0 = \frac{\lambda}{4}$
$n=1 \quad x_1 = \frac{3\lambda}{4}$
$\Rightarrow \boxed{x_1 - x_0 = \frac{\lambda}{2}}$

The distance between two successive nodes is equal to the distance between two successive AN and equal to $\frac{\lambda}{2}$



b)



section in the string when wave passes

slope $= \frac{\Delta y}{\Delta x}$

$$\frac{F_y}{F} = -\text{slope} = -\frac{\partial y(x,t)}{\partial x}$$

The power in a point on a string is:

$$P(x,t) = F_y(x,t)\, v_y(x,t) = -F \frac{\partial y(x,t)}{\partial x} \cdot \frac{\partial y(x,t)}{\partial t}$$

Using the standing wave equation:

$$y(x,t) = 2A \sin kx \sin \omega t$$

$$\frac{\partial y}{\partial x} = 2kA \cos kx \sin \omega t$$

$$\frac{\partial y}{\partial t} = 2\omega A \sin kx \cos \omega t$$

OR

$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$$

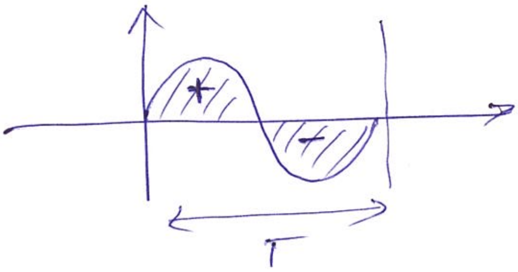
$$\Rightarrow P(x,t) = 4\omega k A^2 \sin kx \cos kx \sin \omega t \cos \omega t$$

$$= \omega k A^2 \underbrace{2\sin kx \cos kx}_{\sin 2kx} \underbrace{2\sin \omega t \cos \omega t}_{\sin 2\omega t}$$

$$\Rightarrow P(x,t) = \omega k A^2 \sin(2kx) \sin(2\omega t)$$

The time average over a period

$$\langle P(x,t) \rangle_T = \omega k A^2 \sin 2kx \underbrace{\langle \sin(2\omega t) \rangle_T}_{0}$$



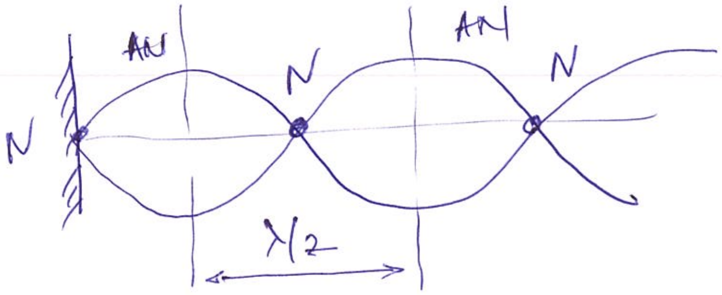
! integral signification = area

$$\langle \sin(2\omega t) \rangle_T = \frac{1}{T} \int_0^T \sin(2\omega t)\, dt = 0.$$

$\Rightarrow \langle P(x,t) \rangle_T = 0$ the average power carried across a point by a standing wave is zero.

(incoming wave and reflected wave carry opposite sign power which cancel out).

③



From the problem

$\Rightarrow \frac{\lambda}{2} = 15$ cm

$\Rightarrow \boxed{\lambda = 30 \text{ cm}}$

ⓐ

The adjacent nodes are also spaced by $\lambda/2$

$\Rightarrow$ the distance between 2 adjacent nodes is also 15 cm.

(b) In an AN the particle oscillates with

amplitude: $A_{SW} = 0{,}850$ cm

period: $T = 0{,}0750$ s

$\Rightarrow$ the amplitude of the standing wave is $2A_0$

where A_0 is the amplitude of the incident and reflected waves

$$\Rightarrow A_0 = \frac{A_{SW}}{2} = \underline{0{,}425 \text{ cm}}$$

The wavelength of the waves is: $\lambda = \underline{30 \text{ cm}}$

the speed: $v = \frac{\lambda}{T} = \frac{30 \text{ cm}}{0{,}0750 \text{ s}} = 400$ cm/s

(c) The standing wave eq. is:

$$y(x,t) = A_{SW} \sin kx \sin \omega t$$

$$\Rightarrow v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = 2A_{SW}\,\omega \sin kx \cos \omega t$$

$\omega = 2\pi/T$

$$\left(v_y(x,t)\right)_{max} = 2A_{SW}\,\omega \sin kx$$

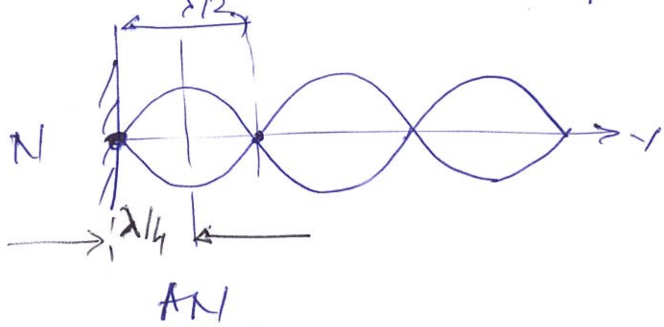
$$= \frac{4A_{SW}\pi}{T} \sin kx$$

For an antinode $\sin kx = \pm 1 \Rightarrow$

$$\left(v_y(x,t)\right)_{max} = \frac{4A_{SW}\pi}{T} = \frac{4\,(0{,}850 \text{ cm}) \cdot 3.14}{0{,}0750 \text{ s}}$$

$$= \underline{142{,}34 \text{ cm/s}}$$

(d) The distance between adjacent N and AN is $\frac{\lambda}{4}$

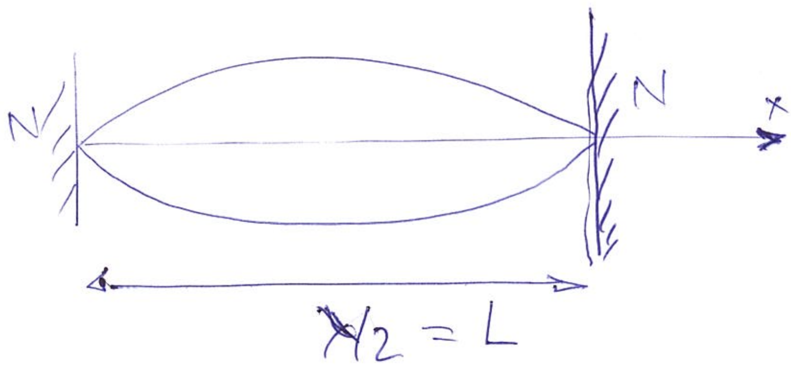


(4) Piano tuner streches the wire with the tension $F = 800\ N$

wire long of: $L = 0{,}4\ m$
wire mass: $m = 3g$

$\Rightarrow$ linear mass density $\mu = \frac{m}{L} = 7{,}5 \cdot 10^{-3} \frac{kg}{m}$

To calculate the frequency of the fundamental mode:



$L = \frac{\lambda_1}{2}$

$\Rightarrow \lambda_1 = 2L$

but $\lambda_1 = \frac{v}{f_1} \Rightarrow f_1 = \frac{v}{\lambda} = \frac{v}{2L}$

the wave velocity in the string is: $v = \sqrt{\frac{F}{\mu}}$

$\Rightarrow \boxed{f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}}$ $\quad f_1 = \frac{1}{2 \cdot 0{,}4}\sqrt{\frac{800}{7{,}5 \cdot 10^{-3}}}$

$\boxed{f_1 = 408{,}24\ Hz}$

The freqvency of higher order harmonics:

$$f_n = n f_1 \qquad n = 2, 3, \ldots$$

If the hearing treshold is 10.000 Hz

$$\Rightarrow \quad n = \frac{10.000}{f_1} = \frac{10000}{408} \approx 24$$

$\Rightarrow$ the higher harmonic heard correspond to $\underline{n = 24}$.

(5) Series of freqvencies:

$$150, 225, 300, 375 \ldots = n \cdot 75 \text{ Hz}$$

$$n = 1, 2 \ldots$$

$\Rightarrow$ the missing freqvency below 400 Hz is
$f_1 = 75$ Hz.

The 7^{th} harmonic $\quad n = 7 \Rightarrow f_7 = 7 \cdot 75 = \underline{525 \text{ Hz}}$

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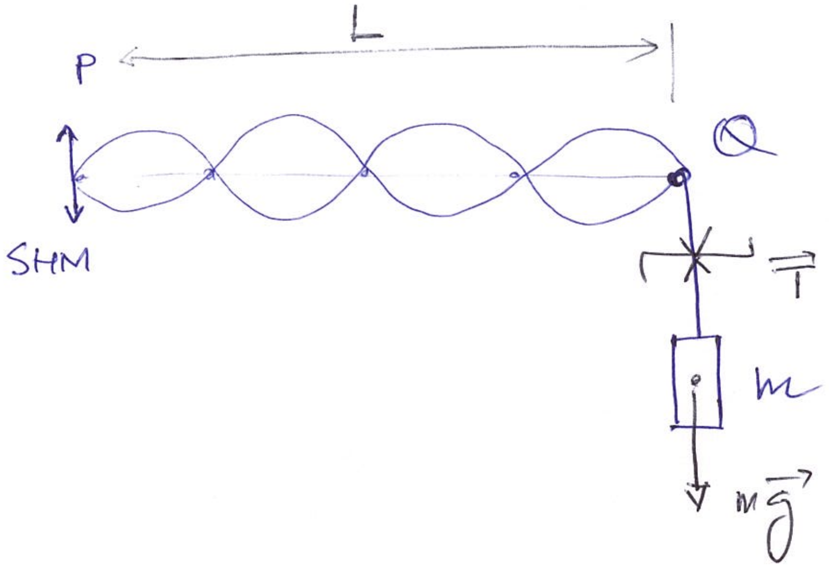

the tension in the wire

$T = mg$

for the 4th harmonic:

$$L = 4\frac{\lambda}{2} = 2\lambda \Rightarrow \lambda_4 = \frac{L}{2}$$

$$\lambda_4 = \frac{v}{f_4} \Rightarrow f_4 = \frac{v}{\lambda_4} = \frac{2v}{L}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

$$\Rightarrow f_4 = \frac{2}{L}\sqrt{\frac{mg}{\mu}} \Rightarrow \boxed{m = \frac{Lf^2\mu}{4g}}$$

$$m = \frac{1,2 \cdot 120^2 \cdot 1,6 \cdot 10^{-3}}{4 \cdot 10} = 0,69\,\text{kg}$$

(b) if $m = 1\,\text{kg} \Rightarrow v = \sqrt{\frac{mg}{\mu}}$

$$f = \frac{v}{\lambda} \Rightarrow \lambda = \frac{v}{f} = \frac{\sqrt{\frac{1 \cdot 9,8}{1,6 \cdot 10^{-3}}}}{120} \simeq 0,65\,\text{m}$$

$$L = n\frac{\lambda}{2} \Rightarrow n = \frac{2L}{\lambda} = \frac{2 \cdot 1,2}{0,65} \simeq 3$$

$\Rightarrow$ one can stabilize the 3rd mode.